

# Disformal invariance of cosmological perturbations in a generalized class of Horndeski theories

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**Abstract.** It is known that Horndeski theories can be transformed to a sub-class of Gleyzes-Langlois-Piazza-Vernizzi (GLPV) theories under the disformal transformation of the metric  $g_{\mu\nu} \rightarrow \Omega^2(\phi)g_{\mu\nu} + \Gamma(\phi, X)\nabla_\mu\phi\nabla_\nu\phi$ , where  $\Omega$  is a function of a scalar field  $\phi$  and  $\Gamma$  is another function depending on both  $\phi$  and  $X = g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$ . We show that, with the choice of unitary gauge, both curvature and tensor perturbations on the flat isotropic cosmological background are generally invariant under the disformal transformation. By means of the effective field theories encompassing Horndeski and GLPV theories, we obtain the second-order actions of scalar/tensor perturbations and present the relations for physical quantities between the two frames. The invariance of the inflationary power spectra under the disformal transformation is explicitly proved up to next-to-leading order in slow-roll. In particular, we identify the existence of the Einstein frame in which the tensor power spectrum is of the same form as that in General Relativity and derive the condition under which the spectrum of gravitational waves in GLPV theories is red-tilted.

**Keywords:** Cosmology of theories beyond the SM, Inflation

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## 1 Introduction

The observational evidence of inflation and dark energy has pushed forward the idea that some scalar degree of freedom beyond the realms of General Relativity (GR) and the standard model of particle physics may be responsible for the two phases of cosmic accelerations [1]. One of the well known modified gravitational theories is Brans-Dicke (BD) theory [2], in which a scalar field  $\phi$  couples to the Ricci scalar  $R$  in the form  $\phi R$ . If we allow the presence of a scalar-field potential, BD theory can accommodate the metric  $f(R)$  gravity as a specific case (the BD parameter  $\omega_{\text{BD}} = 0$  [3, 4]). The Starobinsky model  $f(R) = R + R^2/(6M^2)$  gives rise to inflation due to the dominance of the  $R^2$  term [5]. We also have dark energy models constructed in the framework of  $f(R)$  gravity [6, 7] and BD theory [8].

There are other modified gravitational theories like Galileons [9, 10] in which the field kinetic term  $X = g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$  couples to the Ricci scalar and the Einstein tensor (where  $\nabla_\mu$  represents a covariant derivative). In such theories the cosmic acceleration can be driven by the field kinetic energy even without a scalar potential [11–13]. The Lagrangian of covariant Galileons is constructed to keep the equations of motion up to second order, while recovering

the Galilean symmetry  $\nabla_\mu \phi \rightarrow \nabla_\mu \phi + b_\mu$  in the limit of Minkowski space-time [10]. Without imposing the Galilean symmetry, it is possible to obtain the Lagrangian of most general scalar-tensor theories with second-order equations of motion in generic space-time [14–16]. In fact, this was first derived by Horndeski in 1973 [17].

Horndeski theories encompass a wide variety of gravitational theories including BD theory. In BD theory the conformal transformation of the metric  $g_{\mu\nu} \rightarrow \Omega^2(\phi)g_{\mu\nu}$ , where  $\Omega$  is a function of  $\phi$ , can give rise to a metric frame (dubbed Einstein frame) in which the field  $\phi$  does not have a direct coupling to the Ricci scalar  $\hat{R}$  [18–20]. The situation is more involved in Horndeski theories, but it was shown in Refs. [21, 22] that the so-called disformal transformation in the form  $g_{\mu\nu} \rightarrow \Omega^2(\phi)g_{\mu\nu} + \Gamma(\phi)\nabla_\mu \phi \nabla_\nu \phi$  [23] preserves the structure of the original Horndeski action. Then, it should be possible to identify the Einstein frame in which the field does not have a direct coupling with  $\hat{R}$ .

Recently, Gleyzes, Langlois, Piazza, and Vernizzi (GLPV) [24] proposed a generalized class of Horndeski theories with second-order equations of motion on the flat Friedmann-Lemaître-Robertson-Walker (FLRW) background. According to the Hamiltonian analysis based on linear cosmological perturbations, GLPV theories have one scalar degree of freedom without ghost-like Ostrogradski instabilities [24–27]. In Ref. [26] it was shown that the structure of the GLPV action in unitary gauge is preserved under the disformal transformation  $g_{\mu\nu} \rightarrow \Omega^2(\phi)g_{\mu\nu} + \Gamma(\phi, X)\nabla_\mu \phi \nabla_\nu \phi$ . Hence the dependence of the function  $\Gamma$  on  $X$  generates terms absent in Horndeski theories.

The disformal transformation  $g_{\mu\nu} \rightarrow \Omega^2(\phi)g_{\mu\nu} + \Gamma(\phi, X)\nabla_\mu \phi \nabla_\nu \phi$  is very useful to understand the relation between Horndeski and GLPV theories. As we will see in Sec. 3, the disformal transformation of Horndeski theories gives rise to a sub-class of GLPV theories satisfying one additional condition. Conversely, the transformation from GLPV theories to Horndeski theories demands that the factor  $\Gamma$  obeys two different conditions simultaneously [26]. Thus the full Horndeski and GLPV theories are not equivalent to each other, but the two non-Horndeski pieces in the GLPV action separately arise from a subset of the Horndeski action under the disformal transformation.

In BD and non-minimally coupled theories, it is known that both scalar and tensor perturbations are invariant under the conformal transformation [28–34] (see also Refs. [35, 36]). In fact, this equivalence was used for the computation of the spectral indices of the primordial power spectra and the tensor-to-scalar ratio to confront non-minimally coupled inflationary models with observations [37–39] (e.g., Higgs inflation [40, 41]). The invariance of cosmological perturbations under the disformal transformation  $g_{\mu\nu} \rightarrow \Omega^2(\phi)g_{\mu\nu} + \Gamma(\phi)\nabla_\mu \phi \nabla_\nu \phi$  in Horndeski theories was recently proved in Ref. [42]. In this paper, we show the frame independence of curvature and tensor perturbations on the flat FLRW background under the more general disformal transformation  $g_{\mu\nu} \rightarrow \Omega^2(\phi)g_{\mu\nu} + \Gamma(\phi, X)\nabla_\mu \phi \nabla_\nu \phi$ .

By means of the effective field theory (EFT) of cosmological perturbations developed in Refs. [43–49], we obtain the second-order actions of scalar and tensor perturbations by choosing the unitary gauge. This procedure is closely related to the analysis in Refs. [49–51], but the background lapse dependence is explicitly taken into account for the quantities associated with linear perturbations. The latter treatment is important for understanding the relation between the quantities in the two frames connected by the disformal transformation.

In the EFT approach we also derive the primordial power spectra of scalar and tensor perturbations generated during inflation up to next-to-leading order in slow-roll. The invariance of inflationary observables (such as the spectral indices and the tensor-to-scalar ratio) under the disformal transformation is explicitly shown by paying particular attention

to the change of quantities associated with the perturbation equations of motion. Moreover, we show the existence of the Einstein frame in which the next-to-leading order tensor power spectrum is of the same form as that in GR. We also study the background equations of motion in the Einstein frame and derive the condition under which the inflationary tensor power spectrum is red-tilted in GLPV theories.

This paper is organized as follows. In Sec. 2 we show the general invariance of curvature and tensor perturbations under the disformal transformation  $g_{\mu\nu} \rightarrow \Omega^2(\phi)g_{\mu\nu} + \Gamma(\phi, X)\nabla_\mu\phi\nabla_\nu\phi$  in unitary gauge. In Sec. 3 we discuss how the structure of the GLPV action is preserved under the disformal transformation. In Sec. 4 the second-order actions of scalar and tensor perturbations are derived in the EFT approach encompassing both Horndeski and GLPV theories. In Sec. 5 we present explicit relations between the quantities associated with the background and perturbations in the two frames linked through the disformal transformation by considering GLPV theories. In Sec. 6 we apply the results in Sec. 4 to the derivation of the inflationary power spectra up to next-to-leading order in slow-roll and show their invariance under the disformal transformation. In Sec. 7 we identify the existence of the Einstein frame by appropriately choosing the functions  $\Omega$  and  $\Gamma$ . Sec. 8 is devoted to conclusions.

## 2 Disformal transformation

We begin with the line element based on the Arnowitt-Deser-Misner (ADM) formalism [52] given by

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad (2.1)$$

where  $N$  is the lapse,  $N^i$  is the shift vector,  $g_{\mu\nu}$  and  $h_{ij}$  are the four-dimensional and three-dimensional metrics respectively. Throughout the paper, Greek and Latin indices represent components in space-time and in a three-dimensional space-adapted basis, respectively. The perturbed line element on the flat FLRW background is characterized by [53]

$$ds^2 = -(1 + 2A)dt^2 + 2\psi_{|i}dt dx^i + a^2(t) [(1 + 2\zeta)\delta_{ij} + \gamma_{ij} + 2E_{|ij}] dx^i dx^j, \quad (2.2)$$

where  $a(t)$  is the scale factor that depends on the cosmic time  $t$ , the lower index “ $|i$ ” denotes the covariant derivative with respect to the three-dimensional metric  $h_{ij}$ , and  $A, \psi, \zeta, E$  are the scalar metric perturbations and  $\gamma_{ij}$  is the tensor perturbation. Comparing Eq. (2.1) with Eq. (2.2), there is the correspondence  $1 + 2A = N^2 - h_{ij}N^i N^j$ ,  $\psi_{|i} = h_{ij}N^j$ , and

$$h_{ij} = a^2(t)q_{ij}, \quad \text{where} \quad q_{ij} \equiv (1 + 2\zeta)\delta_{ij} + \gamma_{ij} + 2E_{|ij}. \quad (2.3)$$

For the line element (2.1) we perform the following disformal transformation

$$\hat{g}_{\mu\nu} = \Omega^2(\phi)g_{\mu\nu} + \Gamma(\phi, X)\nabla_\mu\phi\nabla_\nu\phi, \quad (2.4)$$

where  $\Omega(\phi)$  is a function of a scalar field  $\phi$ , and  $\Gamma(\phi, X)$  is a function that depends on  $\phi$  and its kinetic energy  $X = g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$ . In the following we use a hat for the quantities in the transformed frame. We choose the unitary gauge in which  $\phi$  depends on the time  $t$  alone, i.e.,

$$\phi = \phi(t). \quad (2.5)$$

In this case the field kinetic energy is given by  $X = -N^{-2}\dot{\phi}^2$ , where a dot represents a derivative with respect to  $t$ . Hence the dependence on  $\phi$  and  $X$  in  $\Gamma$  can be interpreted as that on  $t$  and  $N$ . The line element in the transformed frame reads

$$\begin{aligned} d\hat{s}^2 = \hat{g}_{\mu\nu}dx^\mu dx^\nu &= -N^2(\Omega^2 + \Gamma X)dt^2 + \Omega^2 h_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \\ &= -\hat{N}^2 dt^2 + \hat{h}_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \end{aligned} \quad (2.6)$$

where in the second line  $\hat{N}$  and  $\hat{h}_{ij}$  are given, respectively, by

$$\hat{N} = N\sqrt{\Omega^2 + \Gamma X}, \quad (2.7)$$

$$\hat{h}_{ij} = \Omega^2 h_{ij}. \quad (2.8)$$

In unitary gauge the conformal factor  $\Omega(\phi)$  depends on  $t$  but not on  $x^i$ , so we can introduce the scale factor in the transformed frame:

$$\hat{a}(t) = \Omega a(t). \quad (2.9)$$

Using Eqs. (2.3), (2.8), and (2.9), the line element (2.6) can be expressed as

$$d\hat{s}^2 = -\hat{N}^2 dt^2 + \hat{a}^2(t) q_{ij}(dx^i + N^i dt)(dx^j + N^j dt). \quad (2.10)$$

Then the three-dimensional tensor  $q_{ij}$  and the shift  $N^i$  are invariant under the disformal transformation, such that

$$\hat{\zeta} = \zeta, \quad (2.11)$$

$$\hat{\gamma}_{ij} = \gamma_{ij}, \quad (2.12)$$

$$\hat{N}^i = N^i, \quad (2.13)$$

and  $\hat{E} = E$ . In unitary gauge (2.5) where the field perturbation  $\delta\phi$  vanishes, the scalar perturbation  $\zeta$  itself is a gauge-invariant quantity [54]. Thus, from Eqs. (2.11) and (2.12), both the curvature perturbation  $\zeta$  and the tensor perturbation  $\gamma_{ij}$  are invariant under the disformal transformation (2.4). This is the generalization of the results of Ref. [42] in which the same disformal invariance was shown for the transformation  $\hat{g}_{\mu\nu} = \Omega^2(\phi)g_{\mu\nu} + \Gamma(\phi)\nabla_\mu\phi\nabla_\nu\phi$ .

While we have shown the invariance of  $\zeta$  and  $\gamma_{ij}$  under the disformal transformation, it remains to see the transformation properties for quantities appearing in the background and perturbation equations of motion. In GLPV theories we shall address this problem in Secs. 3 and 5. Derivation of the relations for quantities in the two different frames is particularly important to identify the tensor and scalar power spectra in the Einstein frame. This issue is addressed in Sec. 7.

### 3 Disformal transformation in GLPV theories

The four-dimensional Lagrangian of the most general scalar-tensor theories with second-order equations of motion (Horndeski theories [17]) is given by the action [14, 15]

$$S = \int d^4x \sqrt{-g} L, \quad (3.1)$$

where  $g$  is the determinant of the metric  $g_{\mu\nu}$ , and

$$L = G_2(\phi, X) + G_3(\phi, X)\Box\phi + G_4(\phi, X)R - 2G_{4,X}(\phi, X)[(\Box\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}] \\ + G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)[(\Box\phi)^3 - 3(\Box\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}_{;\sigma}], \quad (3.2)$$

where a semicolon represents a covariant derivative with  $\Box\phi \equiv (g^{\mu\nu}\phi_{;\nu})_{;\mu}$ ,  $R$  is the Ricci scalar, and  $G_{\mu\nu}$  is the Einstein tensor. The four functions  $G_i$  ( $i = 2, 3, 4, 5$ ) depend on  $\phi$  and  $X$  with the partial derivatives  $G_{i,X} \equiv \partial G_i / \partial X$  and  $G_{i,\phi} \equiv \partial G_i / \partial \phi$ .

The Horndeski Lagrangian (3.2) can be reformulated by using geometric scalar quantities appearing in the ADM formalism [49]. Defining the extrinsic curvature as  $K_{\mu\nu} = h_{\mu}^{\lambda}n_{\nu;\lambda}$ , where  $n_{\mu} = (-N, 0, 0, 0)$  is a unit vector orthogonal to the constant  $t$  hypersurfaces  $\Sigma_t$ , we can construct the following scalar quantities

$$K \equiv K^{\mu}_{\mu}, \quad \mathcal{S} \equiv K_{\mu\nu}K^{\mu\nu}. \quad (3.3)$$

The three-dimensional Ricci tensor  $\mathcal{R}_{\mu\nu} = {}^{(3)}R_{\mu\nu}$  (intrinsic curvature) characterizes the internal geometry of  $\Sigma_t$ . The scalar quantities constructed from  $\mathcal{R}_{\mu\nu}$  and  $K_{\mu\nu}$  are given by

$$\mathcal{R} \equiv \mathcal{R}^{\mu}_{\mu}, \quad \mathcal{U} \equiv \mathcal{R}_{\mu\nu}K^{\mu\nu}, \quad \mathcal{Z} \equiv \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}. \quad (3.4)$$

Choosing the unitary gauge on the flat FLRW background, the dependence on  $\phi$  and  $X$  in the functions  $G_i$  can be interpreted as that on  $t$  and  $N$ . Expressing the scalar quantities like  $\Box\phi$  and  $R$  in terms of the three-dimensional geometric scalars mentioned above, the Horndeski Lagrangian (3.2) is equivalent to [24, 49, 55]

$$L = A_2(N, t) + A_3(N, t)K + A_4(N, t)(K^2 - \mathcal{S}) + B_4(N, t)\mathcal{R} \\ + A_5(N, t)K_3 + B_5(N, t)(\mathcal{U} - K\mathcal{R}/2), \quad (3.5)$$

where

$$K_3 = K^3 - 3K\mathcal{S} + 2K_{\mu\nu}K^{\mu\lambda}K^{\nu}_{\lambda}, \quad (3.6)$$

and

$$A_2 = G_2 - XF_{3,\phi}, \quad A_3 = 2(-X)^{3/2}F_{3,X} - 2\sqrt{-X}G_{4,\phi}, \\ A_4 = -G_4 + 2XG_{4,X} + XG_{5,\phi}/2, \quad B_4 = G_4 + X(G_{5,\phi} - F_{5,\phi})/2, \\ A_5 = -(-X)^{3/2}G_{5,X}/3, \quad B_5 = -\sqrt{-X}F_5. \quad (3.7)$$

Here,  $F_3(\phi, X)$  and  $F_5(\phi, X)$  are auxiliary functions satisfying  $G_3 = F_3 + 2XF_{3,X}$  and  $G_{5,X} = F_5/(2X) + F_{5,X}$ . From Eq. (3.7) the following two relations hold

$$A_4 = 2XB_{4,X} - B_4, \quad A_5 = -XB_{5,X}/3. \quad (3.8)$$

The GLPV theories correspond to the Lagrangian (3.5) without the particular relations (3.8). On the flat FLRW background the function  $K_3$  can be expressed in terms of  $K$  and  $\mathcal{S}$  [49], so the GLPV Lagrangian depends on  $N, t, K, \mathcal{S}, \mathcal{R}, \mathcal{U}$  but not on  $\mathcal{Z}$ . The dependence on  $\mathcal{Z}$  arises for the theories with spatial derivatives higher than second order [56–58], e.g., Horava-Lifshitz gravity [59].

Under the disformal transformation (2.4) the lapse  $\hat{N}$  and the three-dimensional metric  $\hat{h}_{ij}$  in the line element (2.6) are given by Eqs. (2.7) and (2.8) respectively, so the volume element is transformed as [26]

$$\sqrt{-\hat{g}} = \sqrt{-g}\Omega^3\alpha, \quad (3.9)$$

where

$$\alpha \equiv \frac{\hat{N}}{N} = \sqrt{\Omega^2 + \Gamma X}. \quad (3.10)$$

In unitary gauge (2.5), the conformal factor  $\Omega(\phi)$  depends on  $t$  but not on  $N$ . The extrinsic curvature in the transformed frame is given by  $\hat{K}_{ij} = (\partial \hat{h}_{ij} / \partial t - \hat{N}_{i|j} - \hat{N}_{j|i}) / (2\hat{N})$ . On using Eqs. (2.8) and (2.13), it follows that  $\hat{N}_j = \Omega^2(t)N_j$ . Then the transformation of the extrinsic curvature reads

$$\hat{K}_{ij} = \frac{\Omega^2}{\alpha} \left( K_{ij} + \frac{\omega}{N} h_{ij} \right), \quad (3.11)$$

where

$$\omega \equiv \frac{\dot{\Omega}}{\Omega}. \quad (3.12)$$

The transformation (2.8) of the metric  $h_{ij}$  is the same as the conformal transformation in three dimensions. Hence the three-dimensional Ricci tensor transforms as [60]

$$\hat{\mathcal{R}}_{ij} = \mathcal{R}_{ij} - \nabla_i \nabla_j \ln \Omega - g_{ij} g^{kl} \nabla_k \nabla_l \ln \Omega + (\nabla_i \ln \Omega)(\nabla_j \ln \Omega) - g_{ij} g^{kl} (\nabla_k \ln \Omega)(\nabla_l \ln \Omega). \quad (3.13)$$

Since  $\Omega$  is a function of  $t$  alone in unitary gauge, the spatial derivatives of  $\Omega$  vanish in Eq. (3.13). Then the transformations of  $\mathcal{R}_{ij}$  and  $\mathcal{R}$  are simply given by

$$\hat{\mathcal{R}}_{ij} = \mathcal{R}_{ij}, \quad \hat{\mathcal{R}} = \Omega^{-2} \mathcal{R}. \quad (3.14)$$

Employing the transformation laws (3.9), (3.11), and (3.14) for the action (3.1) with the GLPV Lagrangian (3.5), the action in the transformed frame reads

$$S = \int d^4x \sqrt{-\hat{g}} \hat{L}, \quad (3.15)$$

where

$$\begin{aligned} \hat{L} = & \hat{A}_2(\hat{N}, t) + \hat{A}_3(\hat{N}, t) \hat{K} + \hat{A}_4(\hat{N}, t) (\hat{K}^2 - \hat{\mathcal{S}}) + \hat{B}_4(\hat{N}, t) \hat{\mathcal{R}} \\ & + \hat{A}_5(\hat{N}, t) \hat{K}_3 + \hat{B}_5(\hat{N}, t) \left( \hat{\mathcal{U}} - \hat{K} \hat{\mathcal{R}} / 2 \right), \end{aligned} \quad (3.16)$$

with the coefficients [26]

$$\hat{A}_2 = \frac{1}{\Omega^3 \alpha} \left( A_2 - \frac{3\omega}{N} A_3 + \frac{6\omega^2}{N^2} A_4 - \frac{6\omega^3}{N^3} A_5 \right), \quad (3.17)$$

$$\hat{A}_3 = \frac{1}{\Omega^3} \left( A_3 - \frac{4\omega}{N} A_4 + \frac{6\omega^2}{N^2} A_5 \right), \quad (3.18)$$

$$\hat{A}_4 = \frac{\alpha}{\Omega^3} \left( A_4 - \frac{3\omega}{N} A_5 \right), \quad (3.19)$$

$$\hat{B}_4 = \frac{1}{\Omega \alpha} \left( B_4 + \frac{\omega}{2N} B_5 \right), \quad (3.20)$$

$$\hat{A}_5 = \frac{\alpha^2}{\Omega^3} A_5, \quad (3.21)$$

$$\hat{B}_5 = \frac{1}{\Omega} B_5. \quad (3.22)$$

The structure of the Lagrangian (3.16) is the same as the original GLPV Lagrangian (3.5), so the disformal transformation (2.4) allows the connection between the GLPV theories.

Let us consider Horndeski theories described by the Lagrangian (3.5) satisfying the two conditions (3.8). On using Eqs. (3.19)-(3.22) and the correspondence  $\hat{X} = \alpha^{-2}X$ , we obtain

$$\hat{A}_4 + \hat{B}_4 - 2\hat{X}\hat{B}_{4,\hat{X}} = -\frac{X^2\Gamma_{,X}}{\Omega^2 - X^2\Gamma_{,X}}\hat{A}_4, \quad (3.23)$$

$$\hat{A}_5 + \frac{1}{3}\hat{X}\hat{B}_{5,\hat{X}} = -\frac{X^2\Gamma_{,X}}{\Omega^2 - X^2\Gamma_{,X}}\hat{A}_5. \quad (3.24)$$

If  $\Gamma$  is a function of  $\phi$  alone, it follows that  $\hat{A}_4 + \hat{B}_4 - 2\hat{X}\hat{B}_{4,\hat{X}} = 0$  and  $\hat{A}_5 + \hat{X}\hat{B}_{5,\hat{X}}/3 = 0$ . Hence, as shown in Refs. [21, 22], the disformal transformation of the form  $\hat{g}_{\mu\nu} = \Omega^2(\phi)g_{\mu\nu} + \Gamma(\phi)\nabla_\mu\phi\nabla_\nu\phi$  preserves the structure of the Horndeski action. If  $\Gamma$  depends on both  $\phi$  and  $X$ , Horndeski theories are transformed to a sub-class of GLPV theories obeying the particular relation

$$\frac{\hat{B}_4 - 2\hat{X}\hat{B}_{4,\hat{X}}}{\hat{A}_4} = \frac{\hat{X}\hat{B}_{5,\hat{X}}}{3\hat{A}_5}, \quad (3.25)$$

which follows from Eqs. (3.23) and (3.24). Conversely, the full action of GLPV theories cannot be generally mapped to that in Horndeski theories because the function  $\Gamma$  needs to be chosen to satisfy the two Horndeski conditions simultaneously [26].

## 4 Second-order actions of cosmological perturbations

In the EFT of modified gravity including both Horndeski and GLPV theories, the perturbation equations on the flat FLRW background were derived in Refs. [49, 50]. In these papers the background value of the lapse  $N$  (denoted as  $\bar{N}$ ) is set to 1 after obtaining the background and perturbation equations of motion. Since the lapse is transformed as Eq. (2.7) under the disformal transformation, we do not set  $\bar{N} = 1$  in the following discussion. Since the Lagrangian (3.5) in GLPV theories involves the dependence on  $N, K, \mathcal{S}, \mathcal{R}, \mathcal{U}$  and  $t$ , we expand the following action

$$S = \int d^4x \sqrt{-g} L(N, K, \mathcal{S}, \mathcal{R}, \mathcal{U}; t), \quad (4.1)$$

up to quadratic order in perturbations. For the partial derivatives of  $L$  with respect to scalar quantities, we use the notation like  $L_{,N} \equiv \partial L / \partial N$  and  $L_{,K} \equiv \partial L / \partial K$ .

On the flat FLRW background described by the line element  $ds^2 = -\bar{N}^2 dt^2 + a^2(t)\delta_{ij}dx^i dx^j$ , the geometric ADM quantities are given by

$$\bar{K}_{\mu\nu} = H\bar{h}_{\mu\nu}, \quad \bar{K} = 3H, \quad \bar{\mathcal{S}} = 3H^2, \quad \bar{\mathcal{R}}_{\mu\nu} = 0, \quad \bar{\mathcal{R}} = \bar{\mathcal{U}} = 0, \quad (4.2)$$

where a bar represents background quantities, and  $H$  is the Hubble parameter defined by

$$H \equiv \frac{\dot{a}}{\bar{N}a}. \quad (4.3)$$

We also introduce the following perturbed quantities

$$\delta N = N - \bar{N}, \quad \delta K_{\mu\nu} = K_{\mu\nu} - Hh_{\mu\nu}, \quad \delta K = K - 3H, \quad \delta \mathcal{S} = 2H\delta K + \delta K_\nu^\mu \delta K_\mu^\nu. \quad (4.4)$$



Since the intrinsic curvature  $\mathcal{R}$  vanishes on the background, we can write

$$\mathcal{R} = \delta_1 \mathcal{R} + \delta_2 \mathcal{R}, \quad (4.5)$$

where  $\delta_1 \mathcal{R}$  and  $\delta_2 \mathcal{R}$  are the first-order and second-order perturbations, respectively. The scalar  $\mathcal{U}$  is also a perturbed quantity, which satisfies the following relation (up to a boundary term) [49]

$$\lambda(t) \mathcal{U} = \frac{1}{2} \lambda(t) \mathcal{R} K + \frac{1}{2N} \dot{\lambda}(t) \mathcal{R}, \quad (4.6)$$

where  $\lambda(t)$  is an arbitrary function with respect to  $t$ .

Expanding the action (4.1) up to first and second order in scalar perturbations, we can derive the background and scalar perturbation equations of motion respectively. In order to fix the temporal and spatial transformation vectors associated with coordinate transformations, we choose the unitary gauge

$$\delta\phi = 0, \quad E = 0, \quad (4.7)$$

where the former corresponds to Eq. (2.5).

#### 4.1 Background equations

Following the same procedure as that given in Refs. [49, 50], the first-order action of scalar perturbations reduces to  $S^{(1)} = \int d^4x \mathcal{L}_1$  with

$$\mathcal{L}_1 = a^3 (\bar{L} + \bar{N} L_{,N} - 3H\mathcal{F}) \delta N + \bar{N} \left( \bar{L} - \frac{\dot{\mathcal{F}}}{\bar{N}} - 3H\mathcal{F} \right) \delta\sqrt{h} + \bar{N} a^3 \mathcal{E} \delta_1 \mathcal{R}, \quad (4.8)$$

where  $h$  is the determinant of the three-dimensional metric  $h_{ij}$ , and

$$\mathcal{F} \equiv L_{,K} + 2HL_{,S}, \quad (4.9)$$

$$\mathcal{E} \equiv L_{,\mathcal{R}} + \frac{\dot{L}_{,\mathcal{U}}}{2\bar{N}} + \frac{3}{2} HL_{,\mathcal{U}}. \quad (4.10)$$

The last term of Eq. (4.8) is a total derivative irrelevant to the dynamics. Varying the action  $S^{(1)}$  with respect to  $\delta N$  and  $\delta\sqrt{h}$ , we obtain the background equations of motion

$$\bar{L} + \bar{N} L_{,N} - 3H\mathcal{F} = 0, \quad (4.11)$$

$$\bar{L} - \frac{\dot{\mathcal{F}}}{\bar{N}} - 3H\mathcal{F} = 0, \quad (4.12)$$

respectively.

#### 4.2 Second-order action of scalar perturbations

Expanding the action (4.1) up to quadratic order in scalar perturbations, we obtain the second-order action  $S^{(2)} = \int d^4x \mathcal{L}_2$  with

$$\begin{aligned} \mathcal{L}_2 = a^3 \bar{N} & \left[ \left\{ \frac{L_{,N}}{\bar{N}} + \frac{1}{2} L_{,NN} - \frac{3H}{\bar{N}} \left( \mathcal{W} + \frac{3\mathcal{A}H}{2\bar{N}} + \frac{L_{,S}H}{\bar{N}} \right) \right\} \delta N^2 \right. \\ & + \left\{ \frac{\mathcal{W}}{\bar{N}} (3\dot{\zeta} - \Delta\psi) + \frac{4}{\bar{N}} (3HC - \bar{N}\mathcal{D} - \mathcal{E}) \Delta\zeta \right\} \delta N - (3\mathcal{A} + 2L_{,S}) \frac{\dot{\zeta}}{\bar{N}^2} \Delta\psi \\ & - 12\mathcal{C} \frac{\dot{\zeta}}{\bar{N}} \Delta\zeta + \left( \frac{9}{2} \mathcal{A} + 3L_{,S} \right) \frac{\dot{\zeta}^2}{\bar{N}^2} + 2\mathcal{E} \frac{(\partial\zeta)^2}{a^2} \\ & \left. + \frac{1}{2} (\mathcal{A} + 2L_{,S}) \frac{(\Delta\psi)^2}{\bar{N}^2} + 4 \frac{\mathcal{C}}{\bar{N}} (\Delta\psi)(\Delta\zeta) + 8\mathcal{G} (\Delta\zeta)^2 \right], \quad (4.13) \end{aligned}$$

where  $(\partial\zeta)^2 = \delta^{ij}(\partial_i\zeta)(\partial_j\zeta) = \delta^{ij}(\partial\zeta/\partial x^i)(\partial\zeta/\partial x^j)$ ,  $\Delta = \nabla_i\nabla^i = a^{-2}(t)\delta^{ij}\partial_i\partial_j \equiv a^{-2}(t)\partial^2$ , and

$$\mathcal{A} = L_{,KK} + 4HL_{,KS} + 4H^2L_{,SS}, \quad (4.14)$$

$$\mathcal{C} = L_{,K\mathcal{R}} + 2HL_{,S\mathcal{R}} + \frac{1}{2}L_{,\mathcal{U}} + HL_{,K\mathcal{U}} + 2H^2L_{,S\mathcal{U}}, \quad (4.15)$$

$$\mathcal{D} = L_{,N\mathcal{R}} - \frac{\dot{L}_{,\mathcal{M}}}{2\bar{N}^2} + HL_{,N\mathcal{U}}, \quad (4.16)$$

$$\mathcal{G} = L_{,\mathcal{R}\mathcal{R}} + 2HL_{,\mathcal{R}\mathcal{U}} + H^2L_{,\mathcal{U}\mathcal{U}}, \quad (4.17)$$

$$\mathcal{W} = L_{,KN} + 2HL_{,SN} - \frac{H}{\bar{N}}(3\mathcal{A} + 2L_{,S}). \quad (4.18)$$

On using the fact that the term  $K_3$  in the Lagrangian (3.5) is given by

$$K_3 = 3H(K^2 - \mathcal{S} - 2KH + 2H^2) \quad (4.19)$$

up to quadratic order in perturbations [49], the GLPV theories (3.5) obey the three conditions

$$\mathcal{A} + 2L_{,S} = 0, \quad \mathcal{C} = 0, \quad \mathcal{G} = 0, \quad (4.20)$$

under which the spatial derivatives higher than second order are absent in Eq. (4.13). We shall employ the conditions (4.20) in the following discussion. Varying the action (4.13) with respect to  $\delta N$  and  $\psi$ , we obtain the Hamiltonian and momentum constraints respectively:

$$\left(2L_{,N} + \bar{N}L_{,NN} - 6H\mathcal{W} + \frac{12H^2L_{,S}}{\bar{N}}\right)\delta N + \left(3\dot{\zeta} - \frac{\partial^2\psi}{a^2}\right)\mathcal{W} - 4(\bar{N}\mathcal{D} + \mathcal{E})\frac{\partial^2\zeta}{a^2} = 0, \quad (4.21)$$

$$\mathcal{W}\delta N - 4L_{,S}\frac{\dot{\zeta}}{\bar{N}} = 0. \quad (4.22)$$

Expressing  $\delta N$  and  $\partial^2\psi/a^2$  in terms of  $\dot{\zeta}$  and  $\partial^2\zeta/a^2$  from Eqs. (4.21)-(4.22) and substituting them into Eq. (4.13), the second-order Lagrangian density (4.13) reduces to (up to boundary terms)

$$\mathcal{L}_2 = a^3q_s \left[ \dot{\zeta}^2 - \frac{c_s^2}{a^2}(\partial\zeta)^2 \right], \quad (4.23)$$

where

$$q_s = \frac{2L_{,S}[4L_{,S}(2\bar{N}L_{,N} + \bar{N}^2L_{,NN}) + 3(\bar{N}\mathcal{W} - 4HL_{,S})^2]}{\bar{N}^3\mathcal{W}^2}, \quad (4.24)$$

$$c_s^2 = \frac{2\bar{N}}{q_s} \left( \frac{\dot{\mathcal{M}}}{\bar{N}} + H\mathcal{M} - \mathcal{E} \right), \quad (4.25)$$

and

$$\mathcal{M} = \frac{4L_{,S}(\bar{N}\mathcal{D} + \mathcal{E})}{\bar{N}\mathcal{W}} = \frac{4L_{,S}}{\bar{N}\mathcal{W}} \left( L_{,\mathcal{R}} + \bar{N}L_{,\mathcal{R}N} + \frac{3}{2}HL_{,\mathcal{U}} + \bar{N}HL_{,N\mathcal{U}} \right). \quad (4.26)$$

From the Lagrangian density (4.23) we obtain the equation of motion for scalar perturbations

$$\frac{d}{dt}(a^3q_s\dot{\zeta}) - aq_sc_s^2\partial^2\zeta = 0, \quad (4.27)$$

which is of second order. In order to avoid ghosts and Laplacian instabilities, we require that  $q_s > 0$  and  $c_s^2 > 0$  respectively. In Sec. 6, we solve Eq. (4.27) for the computation of the power spectrum of curvature perturbations generated during inflation.

### 4.3 Second-order action of tensor perturbations

We derive the second-order action  $S_2^{(h)} = \int d^4x \sqrt{-g} L_2^{(h)}$  for tensor perturbations  $\gamma_{ij}$ . The non-vanishing terms in the second-order Lagrangian  $L_2^{(h)}$  are  $L_{,\mathcal{S}} \delta K_j^i \delta K_i^j$  and  $\mathcal{E}\mathcal{R}$ , where  $\delta K_j^i = \delta^{ik} \dot{\gamma}_{kj} / (2\bar{N})$  and  $\mathcal{R} = \delta^{ik} \delta^{jl} \gamma_{ij} \Delta \gamma_{kl}$  [49, 51, 58]. Then, the second-order Lagrangian density  $\mathcal{L}_2^{(h)} = \sqrt{-g} L_2^{(h)}$  reads

$$\mathcal{L}_2^{(h)} = a^3 q_t \delta^{ik} \delta^{jl} \left( \dot{\gamma}_{ij} \dot{\gamma}_{kl} - \frac{c_t^2}{a^2} \partial \gamma_{ij} \partial \gamma_{kl} \right), \quad (4.28)$$

where

$$q_t = \frac{L_{,\mathcal{S}}}{4\bar{N}}, \quad (4.29)$$

$$c_t^2 = \frac{\bar{N}^2 \mathcal{E}}{L_{,\mathcal{S}}}. \quad (4.30)$$

The resulting equation of motion for tensor perturbations is given by

$$\frac{d}{dt} (a^3 q_t \dot{\gamma}_{ij}) - a q_t c_t^2 \partial^2 \gamma_{ij} = 0, \quad (4.31)$$

which is of the same form as Eq. (4.27) apart from the difference of the coefficients  $q_t$  and  $c_t$ .

## 5 Relations between the two frames connected by the disformal transformation

In GLPV theories, we shall show the relations between the quantities in the two frames linked through the disformal transformation. Under the disformal transformation of the action (3.1) with the Lagrangian (3.5), we obtain the action (3.15) with  $\hat{L}$  given by Eq. (3.16).

### 5.1 Background quantities

Let us first discuss the transformation of the background equations of motion (4.11)-(4.12). Since  $\hat{a} = \Omega a$ , the Hubble parameter in the new frame, defined by  $\hat{H} = \dot{\hat{a}} / (\hat{N} \hat{a})$ , is related to  $H = \dot{a} / (\bar{N} a)$  as

$$\hat{H} = \frac{1}{\bar{\alpha}} \left( H + \frac{\omega}{\bar{N}} \right), \quad (5.1)$$

where

$$\bar{\alpha} \equiv \frac{\hat{N}}{\bar{N}}. \quad (5.2)$$

In GLPV theories the function  $\hat{\mathcal{F}}$  in the transformed frame is given by  $\hat{\mathcal{F}} = \hat{A}_3 + 4\hat{H}\hat{A}_4 + 6\hat{H}^2\hat{A}_5$ . Using the transformation laws (3.18), (3.19), (3.21) and (5.1), it follows that

$$\hat{\mathcal{F}} = \frac{1}{\Omega^3} \mathcal{F}, \quad (5.3)$$

where  $\mathcal{F} = A_3 + 4HA_4 + 6H^2A_5$ . Similarly, the transformation of the background Lagrangian  $\bar{L} = A_2 + 3HA_3 + 6H^2A_4 + 6H^3A_5$  is

$$\hat{\bar{L}} = \frac{1}{\Omega^3 \bar{\alpha}} \bar{L}. \quad (5.4)$$

Taking the  $\hat{N}$  derivative of the coefficients (3.17)-(3.19) and (3.21), we find that the background value of  $\hat{L}_{,\hat{N}}$  is given by

$$\hat{L}_{,\hat{N}} = \frac{\bar{\beta}}{\Omega^3 \bar{\alpha}} \left[ L_{,N} - \bar{\mu}(\bar{L} - 3H\mathcal{F}) + \frac{3\omega\mathcal{F}}{\bar{N}^2}(1 + \bar{\mu}\bar{N}) \right], \quad (5.5)$$

where, in the square bracket of Eq. (5.5),  $N$  is replaced by  $\bar{N}$  after taking the  $N$  derivative, and

$$\bar{\beta} \equiv \frac{\partial N}{\partial \hat{N}} \Big|_{\hat{N}=\bar{N}}, \quad \bar{\mu} \equiv \frac{1}{\alpha} \frac{\partial \alpha}{\partial N} \Big|_{N=\bar{N}}. \quad (5.6)$$

From Eq. (2.7) there is the following relation

$$\bar{\beta} = \left( \frac{\partial \hat{N}}{\partial N} \right)^{-1} \Big|_{N=\bar{N}} = \frac{2\bar{\alpha}\bar{N}}{2\bar{N}\Omega^2 - \Gamma_{,N}\dot{\phi}^2}, \quad (5.7)$$

so that  $\bar{\mu}$ ,  $\bar{\alpha}$ , and  $\bar{\beta}$  are related with each other, as

$$\bar{\mu}\bar{N} = \frac{1}{\bar{\alpha}\bar{\beta}} - 1. \quad (5.8)$$

On using Eqs. (4.11) and (5.8), Eq. (5.5) reduces to

$$\hat{L}_{,\hat{N}} = \frac{1}{\Omega^3 \bar{\alpha}^2} \left( L_{,N} + \frac{3\omega\mathcal{F}}{\bar{N}^2} \right). \quad (5.9)$$

From Eqs. (5.1), (5.3), (5.4), and (5.9) it follows that

$$\hat{\hat{L}} + \hat{N}\hat{L}_{,\hat{N}} - 3\hat{H}\hat{\mathcal{F}} = \frac{1}{\Omega^3 \bar{\alpha}} (\bar{L} + \bar{N}L_{,N} - 3H\mathcal{F}) = 0, \quad (5.10)$$

$$\hat{\hat{L}} - \frac{\dot{\hat{\mathcal{F}}}}{\hat{\hat{N}}} - 3\hat{H}\hat{\mathcal{F}} = \frac{1}{\Omega^3 \bar{\alpha}} \left( \bar{L} - \frac{\dot{\mathcal{F}}}{\bar{N}} - 3H\mathcal{F} \right) = 0, \quad (5.11)$$

where we used the background Eqs. (4.11)-(4.12) in the original frame. Equations (5.10)-(5.11) are equivalent to those derived by varying the action  $S = \int d^4x \sqrt{-\hat{g}} \hat{L}$  in the transformed frame.

## 5.2 Quantities associated with perturbations

For scalar perturbations the second-order action in the transformed frame is given by  $S^{(2)} = \int d^4x \hat{\mathcal{L}}_2$ , where

$$\hat{\mathcal{L}}_2 = \hat{a}^3 \hat{q}_s \left[ \dot{\hat{\zeta}}^2 - \frac{\hat{c}_s^2}{\hat{a}^2} (\partial \hat{\zeta})^2 \right]. \quad (5.12)$$

Since the curvature perturbation  $\zeta$  is invariant under the disformal transformation, the equivalence between the Lagrangian densities (4.23) and (5.12) implies the following relations

$$\hat{q}_s = \frac{1}{\Omega^3} q_s, \quad (5.13)$$

$$\hat{c}_s^2 = \Omega^2 c_s^2. \quad (5.14)$$

In what follows we shall derive these relations in GLPV theories by explicitly employing the transformation laws (3.17)-(3.22) for the background variables appearing on the r.h.s. of Eqs. (4.24)-(4.25).

The quantities  $L_{,S} = -A_4 - 3HA_5$  and  $\mathcal{W} = A_{3,N} + 4HA_{4,N} + 6H^2A_{5,N} - 4H(A_4 + 3HA_5)/\bar{N}$  transform, respectively, as

$$\hat{L}_{,\hat{S}} = \frac{\bar{\alpha}}{\Omega^3} L_{,S}, \quad \hat{\mathcal{W}} = \frac{\bar{\beta}}{\Omega^3} \mathcal{W}. \quad (5.15)$$

We recall that the transformation of  $L_{,N}$  is given by Eq. (5.9). For the computation of the quantity  $\hat{L}_{,\hat{N}\hat{N}}$ , we take the second derivatives of Eqs. (3.17)-(3.19) and (3.21) with respect to  $\hat{N}$  and then substitute the relations  $A_2 = 6H^2A_4 + 12H^3A_5 - \bar{N}L_{,N}$  [which comes from Eq. (4.11)],  $A_{2,N} = L_{,N} - 3HA_{3,N} - 6H^2A_{4,N} - 6H^3A_{5,N}$ ,  $A_{2,NN} = L_{,NN} - 3HA_{3,NN} - 6H^2A_{4,NN} - 6H^3A_{5,NN}$ ,  $A_4 = -L_{,S} - 3HA_5$ ,  $A_{3,N} = \mathcal{W} - 4HA_{4,N} - 6H^2A_{5,N} - 4L_{,S}H/\bar{N}$ , and Eq. (5.8) into the expression of  $\hat{L}_{,\hat{N}\hat{N}}$ . This process leads to

$$\begin{aligned} \hat{L}_{,\hat{N}\hat{N}} = \frac{1}{\Omega^3 \bar{\alpha}^3 \bar{N}^4} & \left[ \bar{\alpha} \bar{\beta} \bar{N}^3 \{ \bar{\alpha} \bar{\beta} \bar{N} L_{,NN} - 6H(\bar{\alpha} \bar{\beta} - 1) \mathcal{W} \} + 2(\bar{\alpha}^2 \bar{\beta}^2 - 1) \bar{N}^2 (\bar{N} L_{,N} + 6H^2 L_{,S}) \right. \\ & \left. - 6\omega (\bar{N} \mathcal{F} + 4\bar{N} H L_{,S} - \bar{\alpha} \bar{\beta} \bar{N}^2 \mathcal{W} + 2L_{,S} \omega) \right]. \end{aligned} \quad (5.16)$$

On using Eqs. (5.1), (5.9), (5.15), and (5.16), it follows that

$$\begin{aligned} & 4\hat{L}_{,\hat{S}}(2\hat{N}\hat{L}_{,\hat{N}} + \hat{N}^2\hat{L}_{,\hat{N}\hat{N}}) + 3(\hat{N}\hat{\mathcal{W}} - 4\hat{H}\hat{L}_{,\hat{S}})^2 \\ & = \frac{\bar{\alpha}^2 \bar{\beta}^2}{\Omega^6} [4L_{,S}(2\bar{N}L_{,N} + \bar{N}^2L_{,NN}) + 3(\bar{N}\mathcal{W} - 4HL_{,S})^2]. \end{aligned} \quad (5.17)$$

From Eqs. (5.15) and (5.17) the quantity  $q_s$  defined by Eq. (4.24) indeed obeys the transformation law (5.13).

In GLPV theories the quantities  $\mathcal{E}$  and  $\mathcal{M}$  are given, respectively, by  $\mathcal{E} = B_4 + \dot{B}_5/(2\bar{N})$  and  $\mathcal{M} = 4L_{,S}(B_4 + \bar{N}B_{4,N} - H\bar{N}B_{5,N}/2)/(\bar{N}\mathcal{W})$ . Employing Eqs. (3.20), (3.22), (5.1), (5.8), and (5.15), we find that  $\mathcal{E}$  and  $\mathcal{M}$  transform as

$$\hat{\mathcal{E}} = \frac{1}{\Omega \bar{\alpha}} \mathcal{E}, \quad (5.18)$$

$$\hat{\mathcal{M}} = \frac{1}{\Omega} \mathcal{M}. \quad (5.19)$$

From the definition (4.25) of  $c_s^2$  together with Eqs. (5.1), (5.13), (5.18), and (5.19), it follows that the scalar propagation speed squared transforms as Eq. (5.14).

The quantities  $q_t$  and  $c_t^2$  appearing in the tensor perturbation equation of motion are given, respectively, by Eqs. (4.29) and (4.30). From Eqs. (5.15) and (5.18) they transform as

$$\hat{q}_t = \frac{1}{\Omega^3} q_t, \quad (5.20)$$

$$\hat{c}_t^2 = \Omega^2 c_t^2. \quad (5.21)$$

These properties also follow from the equivalence of the second-order tensor Lagrangian densities  $\mathcal{L}_2^{(h)}$  and  $\hat{\mathcal{L}}_2^{(h)}$  in the two frames together with the invariance (2.12) of tensor perturbations.

## 6 Next-to-leading order inflationary power spectra

We derive the power spectra of scalar and tensor perturbations generated during inflation up to next-to-leading order in slow-roll and show that they are invariant under the disformal transformation.

We shall consider the quasi de-Sitter background on which the expansion rate defined by

$$h \equiv \bar{N}H = \frac{\dot{a}}{a} \quad (6.1)$$

is nearly constant. In this case the slow-roll parameter

$$\epsilon_h \equiv -\frac{\dot{h}}{h^2} \quad (6.2)$$

is much smaller than 1. The conformal time  $\tau \equiv \int a^{-1} dt$  is approximately given by  $\tau \simeq -1/(ah)$  on the quasi de-Sitter background (where we set the integration constant 0), so the asymptotic past and future correspond to  $\tau \rightarrow -\infty$  and  $\tau \rightarrow 0$ , respectively. If there is a function  $f(\tau)$  that varies slowly during inflation, the expansion of  $f(\tau)$  around a chosen time  $\tau_k$  is [61]

$$f(\tau) \simeq f(\tau_k) \left[ 1 - \delta_f(\tau_k) \ln \frac{\tau}{\tau_k} \right], \quad \delta_f \equiv \frac{\dot{f}}{hf}, \quad (6.3)$$

where  $\delta_f$  is regarded as a slow-roll parameter which is the same order as  $\epsilon_h$ .

### 6.1 The scalar power spectrum

The Lagrangian density for scalar perturbations is given by Eq. (4.23). The curvature perturbation  $\zeta(\mathbf{x}, \tau)$  in real space can be expressed in terms of the Fourier components  $\zeta(k, \tau)$  with the comoving wave number  $\mathbf{k}$ , as

$$\zeta(\mathbf{x}, \tau) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\mathbf{k} \cdot \mathbf{x}} \tilde{\zeta}(\mathbf{k}, \tau), \quad \tilde{\zeta}(\mathbf{k}, \tau) = \zeta(k, \tau) a(\mathbf{k}) + \zeta^*(k, \tau) a^\dagger(-\mathbf{k}), \quad (6.4)$$

where  $a(\mathbf{k})$  and  $a^\dagger(\mathbf{k}')$  are the annihilation and creation operators satisfying the commutation relation  $[a(\mathbf{k}), a^\dagger(\mathbf{k}')] = \delta^{(3)}(\mathbf{k} - \mathbf{k}')$ . A rescaled field defined by

$$v_s(k, \tau) = z_s \zeta(k, \tau), \quad z_s = a\sqrt{2q_s} \quad (6.5)$$

corresponds to a canonical scalar field associated with the quantization procedure. From Eq. (4.27) it follows that

$$v_s'' + \left( c_s^2 k^2 - \frac{z_s''}{z_s} \right) v_s = 0, \quad (6.6)$$

where a prime represents the derivative with respect to  $\tau$ .

We introduce the slow-roll parameters

$$\epsilon_s \equiv \frac{\dot{q}_s}{hq_s}, \quad s_s \equiv \frac{\dot{c}_s}{hc_s}, \quad (6.7)$$

and pick up the terms up to next-to-leading order in slow-roll under the assumption that  $|\epsilon_s|$  and  $|s_s|$  are much smaller than 1 during inflation. The quantity  $z_s''/z_s$  in Eq. (6.6) can be estimated as

$$\frac{z_s''}{z_s} \simeq 2(ah)^2 \left( 1 - \frac{1}{2}\epsilon_h + \frac{3}{4}\epsilon_s \right). \quad (6.8)$$

The sound-horizon crossing corresponds to the epoch characterized by  $c_s k = ah$ . Defining the dimensionless variable

$$y_s \equiv \frac{c_s k}{ah}, \quad (6.9)$$

Eq. (6.6) can be written as

$$(1 - 2\epsilon_h - 2s_s)y_s^2 \frac{d^2 v_s}{dy_s^2} - s_s y_s \frac{dv_s}{dy_s} + \left( y_s^2 - 2 + \epsilon_h - \frac{3}{2}\epsilon_s \right) v_s = 0. \quad (6.10)$$

The solution to this equation is given by

$$v_s = y_s^{(1+s_s)/2} \left\{ \alpha_k H_\nu^{(1)}[(1 + \epsilon_h + s_s)y_s] + \beta_k H_\nu^{(2)}[(1 + \epsilon_h + s_s)y_s] \right\}, \quad (6.11)$$

where  $\alpha_k$  and  $\beta_k$  are integration constants,  $H_\nu^{(1)}(x)$  and  $H_\nu^{(2)}(x)$  are Hankel functions of the first and second kinds respectively, and

$$\nu = \frac{3}{2} + \epsilon_h + \frac{1}{2}\epsilon_s + \frac{3}{2}s_s. \quad (6.12)$$

For the derivation of the solution (6.11), we have ignored the slow-roll parameters higher than second order.

The quantities  $h$ ,  $q_s$ , and  $c_s$  have runnings according to Eq. (6.3), i.e.,  $\delta_f = -\epsilon_h, \epsilon_s, s_s$  for  $f = h, q_s, c_s$ , respectively. Provided that  $|\delta_f| \ll 1$ , the function  $1 - \delta_f \ln(\tau/\tau_k)$  is approximately equivalent to  $y_s^{-\delta_f}$ . Hence the runnings of  $h$ ,  $q_s$ , and  $c_s$  can be quantified as

$$h = h_k y_s^{\epsilon_h}, \quad q_s = q_{sk} y_s^{-\epsilon_s}, \quad c_s = c_{sk} y_s^{-s_s}, \quad (6.13)$$

where the lower indices  $k$  represent the values at  $y_s = 1$ .

The positive-frequency solution satisfying the Wronskian condition  $v_s v_s^{*'} - v_s^* v_s' = i$  in the asymptotic past ( $y_s \rightarrow \infty$ ) corresponds to the coefficients

$$\alpha_k = -\frac{1}{2} \sqrt{\frac{\pi}{c_{sk} k}} \left( 1 + \frac{1}{2}\epsilon_h + \frac{1}{2}s_s \right), \quad \beta_k = 0, \quad (6.14)$$

where we exploited the last relation of Eq. (6.13). Substituting Eq. (6.14) into Eq. (6.11) and using the first and second relations of Eq. (6.13), the solution to the curvature perturbation  $\zeta = v_s/z_s$  long after the sound horizon crossing ( $y_s \rightarrow 0$ ) is given by

$$\zeta(k, 0) = i \frac{2^\nu \Gamma(\nu)}{\sqrt{8\pi q_{sk}}} \frac{1 - \epsilon_h - s_s}{(c_{sk} k)^{3/2}} h_k, \quad (6.15)$$

where  $\Gamma(\nu)$  is the Gamma function.

The power spectrum  $\mathcal{P}_\zeta$  is defined by the vacuum expectation value of the two-point correlation function of  $\zeta$ , as

$$\langle 0 | \tilde{\zeta}(\mathbf{k}_1, 0) \tilde{\zeta}(\mathbf{k}_2, 0) | 0 \rangle = \frac{2\pi^2}{k_1^3} \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2) \mathcal{P}_\zeta(k_1). \quad (6.16)$$

Employing the solution (6.15) and expanding the Gamma function around  $\nu = 3/2$ , we obtain the next-to-leading order scalar power spectrum

$$\mathcal{P}_\zeta(k) = \frac{h^2}{8\pi^2 q_s c_s^3} \left[ 1 - 2(C + 1)\epsilon_h - C\epsilon_s - (3C + 2)s_s \right] \Big|_{c_s k = ah}, \quad (6.17)$$

where  $C = \gamma - 2 + \ln 2 = -0.729637\dots$  ( $\gamma$  is the Euler-Mascheroni constant).

## 6.2 The tensor power spectrum

The next-to-leading order power spectrum of tensor perturbations was recently derived in Ref. [58]. Here, we briefly summarize the result by explicitly taking into account the lapse function  $\bar{N}$ . We first expand  $\gamma_{ij}(\mathbf{x}, \tau)$  into the Fourier series as

$$\gamma_{ij}(\mathbf{x}, \tau) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}} \tilde{\gamma}_{ij}(\mathbf{k}, \tau), \quad \tilde{\gamma}_{ij}(\mathbf{k}, \tau) = \sum_{\lambda=+, \times} \tilde{h}_\lambda(\mathbf{k}, \tau) e_{ij}^{(\lambda)}(\mathbf{k}), \quad (6.18)$$

where  $e_{ij}^{(\lambda)}(\mathbf{k})$  are the transverse and traceless polarization tensors satisfying the normalization  $e_{ij}^{(\lambda)}(\mathbf{k}) e_{ij}^{*(\lambda')}(\mathbf{k}) = \delta_{\lambda\lambda'}$ . The annihilation and creation operators  $a_\lambda(\mathbf{k})$  and  $a_\lambda^\dagger(\mathbf{k}')$  obey the commutation relation  $[a_\lambda(\mathbf{k}), a_{\lambda'}^\dagger(\mathbf{k}')] = \delta_{\lambda\lambda'} \delta^{(3)}(\mathbf{k} - \mathbf{k}')$ . The Fourier mode  $\tilde{h}_\lambda(\mathbf{k}, \tau)$  is expressed in the form

$$\tilde{h}_\lambda(\mathbf{k}, \tau) = h_\lambda(k, \tau) a_\lambda(\mathbf{k}) + h_\lambda^*(k, \tau) a_\lambda^\dagger(-\mathbf{k}). \quad (6.19)$$

Defining a canonically normalized field  $v_\lambda(k, \tau)$  as

$$v_\lambda(k, \tau) \equiv z_t h_\lambda(k, \tau), \quad z_t \equiv a\sqrt{2q_t}, \quad (6.20)$$

Eq. (4.31) reduces to

$$v_\lambda'' + \left( c_t^2 k^2 - \frac{z_t''}{z_t} \right) v_\lambda = 0. \quad (6.21)$$

We can derive the solution to this equation by introducing a dimensionless parameter  $y_t \equiv c_t k / (ah)$ . Following the similar procedure to that performed for scalar perturbations, the solution to  $h_\lambda$  in the regime  $\tau \rightarrow 0$  is given by

$$h_\lambda(k, 0) = i \frac{2^{\nu_t} \Gamma(\nu_t)}{\sqrt{8\pi q_{tk}}} \frac{1 - \epsilon_h - s_t}{(c_{tk} k)^{3/2}} h_k, \quad (6.22)$$

where the lower index  $k$  represents the values at  $y_t = 1$ , and

$$\nu_t = \frac{3}{2} + \epsilon_h + \frac{1}{2}\epsilon_t + \frac{3}{2}s_t, \quad \epsilon_t \equiv \frac{\dot{q}_t}{h q_t}, \quad s_t \equiv \frac{\dot{c}_t}{h c_t}. \quad (6.23)$$

Defining the tensor power spectrum  $\mathcal{P}_h$  as

$$\langle 0 | \tilde{\gamma}_{ij}(\mathbf{k}_1, 0) \tilde{\gamma}_{ij}(\mathbf{k}_2, 0) | 0 \rangle = \frac{2\pi^2}{k_1^3} \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2) \mathcal{P}_h(k_1), \quad (6.24)$$

it follows that

$$\mathcal{P}_h(k) = \frac{h^2}{4\pi^2 q_t c_t^3} \left[ 1 - 2(C+1)\epsilon_h - C\epsilon_t - (3C+2)s_t \right] \Big|_{c_t k = ah}. \quad (6.25)$$

We note that the tensor power spectrum should be evaluated at  $c_t k = ah$ , which is generally different from the moment  $c_s k = ah$  for the sound horizon crossing of scalar perturbations.



### 6.3 Spectral indices and the tensor-to-scalar ratio

The spectral index of scalar perturbations is defined by

$$n_s - 1 = \left. \frac{d \ln \mathcal{P}_\zeta(k)}{d \ln k} \right|_{c_s k = ah}, \quad (6.26)$$

which reduces to  $n_s - 1 = (\dot{\mathcal{P}}_\zeta / \mathcal{P}_\zeta)(dt/d \ln k)|_{c_s k = ah}$ . Taking the time derivative of Eq. (6.17) and using the relation  $d \ln k / dt|_{c_s k = ah} = h_k(1 - \epsilon_h - s_s)$ , we obtain

$$\begin{aligned} n_s - 1 = & -2\epsilon_h - \epsilon_s - 3s_s - 2\epsilon_h^2 - 5\epsilon_h s_s - \epsilon_h \epsilon_s - \epsilon_s s_s - 3s_s^2 \\ & - 2(C + 1)\epsilon_h \eta_h - C\epsilon_s \eta_s - (3C + 2)s_s \mu_s \Big|_{c_s k = ah}, \end{aligned} \quad (6.27)$$

where

$$\eta_h \equiv \frac{\dot{\epsilon}_h}{h\epsilon_h}, \quad \eta_s \equiv \frac{\dot{\epsilon}_s}{h\epsilon_s}, \quad \mu_s \equiv \frac{\dot{s}_s}{hs_s}. \quad (6.28)$$

We also introduce the tensor spectral index

$$n_t = \left. \frac{d \ln \mathcal{P}_h(k)}{d \ln k} \right|_{c_t k = ah}. \quad (6.29)$$

From Eq. (6.25) it follows that

$$\begin{aligned} n_t = & -2\epsilon_h - \epsilon_t - 3s_t - 2\epsilon_h^2 - 5\epsilon_h s_t - \epsilon_h \epsilon_t - \epsilon_t s_t - 3s_t^2 \\ & - 2(C + 1)\epsilon_h \eta_h - C\epsilon_t \eta_t - (3C + 2)s_t \mu_t \Big|_{c_t k = ah}, \end{aligned} \quad (6.30)$$

where

$$\eta_t \equiv \frac{\dot{\epsilon}_t}{h\epsilon_t}, \quad \mu_t \equiv \frac{\dot{s}_t}{hs_t}. \quad (6.31)$$

The scalar and tensor spectra (6.17) and (6.25) are computed at  $c_s k = ah$  and  $c_t k = ah$ , respectively. The moment for the evaluation of the tensor-to-scalar ratio  $r = \mathcal{P}_h(k)/\mathcal{P}_\zeta(k)$  is different depending on the values of  $c_s$  and  $c_t$ .

If  $c_s < c_t$ , the sound horizon crossing for scalar perturbations occurs at  $y_s = c_s k / (ah) = 1$  (denoted as  $\tau = \tau_s$ ), whereas for tensor perturbations it corresponds to the later time characterized by  $y_s = c_s / c_t < 1$  (denoted as  $\tau = \tau_t$ ). Since the tensor perturbation evolves during the epoch  $c_s / c_t < y_s < 1$  and it starts to be frozen in the regime  $y_s < c_s / c_t$ , the tensor-to-scalar ratio should be evaluated at  $y_s = c_s / c_t$ , i.e.,  $c_t k = ah$ .

From Eq. (6.3), any time-dependent function  $f(\tau)$  can be expanded as  $f(\tau) \simeq f(\tau_k)x^{-\delta_f}$  for  $|\delta_f| \ll 1$ , where  $x \equiv k/(ah)$  and  $\tau_k$  is the instant at  $x = 1$ . Since  $x = 1/c_s$  at  $\tau = \tau_s$  and  $x = 1/c_t$  at  $\tau = \tau_t$ , it follows that  $f(\tau_s) \simeq f(\tau_k)c_s^{\delta_f}$  and  $f(\tau_t) \simeq f(\tau_k)c_t^{\delta_f}$ . In this case we have  $f(\tau_s) \simeq [1 + \delta_f \ln(c_s/c_t)] f(\tau_t)$ , so that  $h(\tau_s) = [1 - \epsilon_h \ln(c_s/c_t)] h(\tau_t)$ ,  $q_s(\tau_s) = [1 + \epsilon_s \ln(c_s/c_t)] q_s(\tau_t)$ , and  $c_s(\tau_s) = [1 + s_s \ln(c_s/c_t)] c_s(\tau_t)$ . Substituting these relations into Eq. (6.17), the next-to-leading order scalar power spectrum can be written in the form

$$\mathcal{P}_\zeta(k) = \frac{h^2}{8\pi^2 q_s c_s^3} \left[ 1 - 2(C + 1)\epsilon_h - C\epsilon_s - (3C + 2)s_s - (2\epsilon_h + \epsilon_s + 3s_s) \ln \frac{c_s}{c_t} \right] \Big|_{c_t k = ah}. \quad (6.32)$$

On using Eq. (6.25), the tensor-to-scalar ratio evaluated at  $c_t k = ah$  reads

$$r = 2 \frac{q_s c_s^3}{q_t c_t^3} \left[ 1 - C(\epsilon_t - \epsilon_s) - (3C + 2)(s_t - s_s) + (2\epsilon_h + \epsilon_s + 3s_s) \ln \frac{c_s}{c_t} \right] \Big|_{c_t k = ah}, \quad (6.33)$$

which is valid for  $c_s < c_t$ .

If  $c_s > c_t$ , then the scalar perturbation is frozen at a later epoch relative to the tensor perturbation. Following the similar procedure to that given above, the tensor power spectrum and the tensor-to-scalar ratio at  $c_s k = ah$  are given, respectively, by

$$\mathcal{P}_h(k) = \frac{h^2}{4\pi^2 q_t c_t^3} \left[ 1 - 2(C+1)\epsilon_h - C\epsilon_t - (3C+2)s_t + (2\epsilon_h + \epsilon_t + 3s_t) \ln \frac{c_s}{c_t} \right] \Big|_{c_s k = ah}. \quad (6.34)$$

and

$$r = 2 \frac{q_s c_s^3}{q_t c_t^3} \left[ 1 - C(\epsilon_t - \epsilon_s) - (3C+2)(s_t - s_s) + (2\epsilon_h + \epsilon_t + 3s_t) \ln \frac{c_s}{c_t} \right] \Big|_{c_s k = ah}. \quad (6.35)$$

Compared to Eq. (6.33), the difference arises for the two terms in front of the factor  $\ln(c_s/c_t)$ .

#### 6.4 Equivalence under the disformal transformation

In what follows we show the equivalence of inflationary observables under the disformal transformation by using the relations between the quantities appearing in the scalar and tensor power spectra. Defining the expansion rate  $\hat{h} = \hat{N}\hat{H}$  in the transformed frame, we obtain the following relation from Eq. (5.1):

$$\hat{h} = h(1 + \epsilon_\Omega), \quad \epsilon_\Omega \equiv \frac{\omega}{h} = \frac{\dot{\Omega}}{h\Omega}. \quad (6.36)$$

Then, the slow-roll parameter  $\hat{\epsilon}_h = -\dot{\hat{h}}/\hat{h}^2$  can be expressed as

$$\hat{\epsilon}_h = \epsilon_h - \epsilon_h \epsilon_\Omega - \frac{\dot{\epsilon}_\Omega}{h} + O(\epsilon_h^3). \quad (6.37)$$

On using Eqs. (5.13), (5.14), (5.20) and (5.21), the quantities  $\hat{\epsilon}_s = \dot{\hat{q}}_s/(\hat{h}\hat{q}_s)$ ,  $\hat{s}_s = \dot{\hat{c}}_s/(\hat{h}\hat{c}_s)$ ,  $\hat{\epsilon}_t = \dot{\hat{q}}_t/(\hat{h}\hat{q}_t)$ ,  $\hat{s}_t = \dot{\hat{c}}_t/(\hat{h}\hat{c}_t)$  are given, respectively, by

$$\hat{\epsilon}_s = \epsilon_s - 3\epsilon_\Omega - \epsilon_s \epsilon_\Omega + 3\epsilon_\Omega^2 + O(\epsilon_h^3), \quad \hat{s}_s = s_s + \epsilon_\Omega - s_s \epsilon_\Omega - \epsilon_\Omega^2 + O(\epsilon_h^3), \quad (6.38)$$

$$\hat{\epsilon}_t = \epsilon_t - 3\epsilon_\Omega - \epsilon_t \epsilon_\Omega + 3\epsilon_\Omega^2 + O(\epsilon_h^3), \quad \hat{s}_t = s_t + \epsilon_\Omega - s_t \epsilon_\Omega - \epsilon_\Omega^2 + O(\epsilon_h^3). \quad (6.39)$$

The scalar power spectrum in the transformed frame is given by

$$\hat{\mathcal{P}}_\zeta(k) = \frac{\hat{h}^2}{8\pi^2 \hat{q}_s \hat{c}_s^3} [1 - 2(C+1)\hat{\epsilon}_h - C\hat{\epsilon}_s - (3C+2)\hat{s}_s] \Big|_{\hat{c}_s k = \hat{a}\hat{h}}. \quad (6.40)$$

Substituting Eqs. (5.13), (5.14), (6.36), (6.37), and (6.38) into Eq. (6.40), it follows that

$$\hat{\mathcal{P}}_\zeta(k) = \mathcal{P}_\zeta(k), \quad (6.41)$$

up to next-to-leading order in slow-roll. From Eqs. (2.9), (5.14), and (6.36) the moment  $\hat{c}_s k = \hat{a}\hat{h}$  corresponds to  $c_s k = ah(1 + \epsilon_\Omega)$ , which differs from  $c_s k = ah$  due to the presence of the factor  $\epsilon_\Omega$ . However, this difference does not affect the next-to-leading order power spectrum because the variations of the quantities like  $h, q_s, c_s$  are quantified according to Eq. (6.13). Since the variable  $\hat{y}_s = \hat{c}_s k/(\hat{a}\hat{h})$  is related to  $y_s = c_s k/(ah)$  as  $\hat{y}_s = y_s/(1 + \epsilon_\Omega)$ , the quantities like  $y_s^{\epsilon_h}$  only give rise to second-order slow-roll corrections.

From Eqs. (5.20), (5.21), (6.36), (6.37), and (6.39), it also follows that

$$\hat{\mathcal{P}}_h(k) = \mathcal{P}_h(k), \quad (6.42)$$

up to next-to-leading order.

The spectral index of scalar perturbations in the transformed frame is given by

$$\begin{aligned} \hat{n}_s - 1 = & -2\hat{\epsilon}_h - \hat{\epsilon}_s - 3\hat{s}_s - 2\hat{\epsilon}_h^2 - 5\hat{\epsilon}_h\hat{s}_s - \hat{\epsilon}_h\hat{\epsilon}_s - \hat{\epsilon}_s\hat{s}_s - 3\hat{s}_s^2 \\ & -2(C+1)\hat{\epsilon}_h\hat{\eta}_h - C\hat{\epsilon}_s\hat{\eta}_s - (3C+2)\hat{s}_s\hat{\mu}_s|_{\hat{c}_sk=\hat{a}\hat{h}}. \end{aligned} \quad (6.43)$$

We substitute Eqs. (6.37) and (6.38) into Eq. (6.43) and use the properties that the last two terms of Eq. (6.43) are given by  $-C\hat{\epsilon}_s\hat{\eta}_s = -C\epsilon_s\eta_s + 3C\dot{\epsilon}_\Omega/h + O(\epsilon_h^3)$  and  $-(3C+2)\hat{s}_s\hat{\mu}_s = -(3C+2)s_s\mu_s - (3C+2)\dot{\epsilon}_\Omega/h + O(\epsilon_h^3)$ , respectively. Then the terms involving  $\epsilon_\Omega$  and  $\dot{\epsilon}_\Omega$  vanish, so that we finally obtain

$$\hat{n}_s = n_s. \quad (6.44)$$

Similarly, we can show the equivalence of the tensor spectral index:

$$\hat{n}_t = n_t. \quad (6.45)$$

Substituting Eqs. (5.13), (5.14), (5.20), (5.21), (6.37), (6.38) and (6.39) into Eqs. (6.33) and (6.35), it also follows that

$$\hat{r} = r, \quad (6.46)$$

up to next-to-leading order in slow roll.

## 7 Einstein frame

The action (3.5) of GLPV theories can be transformed to that in the so-called Einstein frame under the disformal transformation. As we will see below, the existence of the Einstein frame is related to a General Relativistic form of the inflationary tensor power spectrum.

### 7.1 Inflationary power spectra in the Einstein frame

From Eq. (6.25) the tensor power spectrum in the transformed frame is given by

$$\hat{\mathcal{P}}_h(k) = \frac{\hat{N}^2 \hat{H}^2}{4\pi^2 \hat{q}_{tk} \hat{c}_{tk}^3} [1 - 2(C+1)\hat{\epsilon}_h - C\hat{\epsilon}_t - (3C+2)\hat{s}_t] |_{\hat{c}_{tk}=\hat{a}\hat{h}}, \quad (7.1)$$

where  $\hat{H} = \dot{\hat{a}}/(\hat{N}\hat{a})$  is the Hubble parameter in the new frame. Note that the lapse  $\hat{N}$  has the same dimension as the tensor propagation speed  $\hat{c}_{tk}$ . Let us consider the case in which the quantities  $\hat{c}_{tk}$  and  $\hat{q}_{tk}$ , after the disformal transformation, are given by

$$\hat{c}_{tk} = \hat{N}, \quad (7.2)$$

$$\hat{q}_{tk} = \frac{M_{\text{pl}}^2}{8\hat{N}}, \quad (7.3)$$

where  $M_{\text{pl}}$  is the reduced Planck mass. On using Eqs. (2.7), (5.20), and (5.21), the choices (7.2) and (7.3) correspond to

$$\Omega^2 = \frac{8q_{tk}c_{tk}}{M_{\text{pl}}^2}, \quad \Gamma = \frac{8q_{tk}c_{tk}}{M_{\text{pl}}^2} \frac{c_{tk}^2 - \bar{N}^2}{\bar{N}^2 X}, \quad (7.4)$$

where  $c_{tk}$  and  $q_{tk}$  should be evaluated on the background such that the kinetic term  $X$  appearing in these quantities is replaced by  $\bar{X} = -\bar{N}^{-2}\dot{\phi}^2$ . For the specific case in which  $\Omega^2$  is equivalent to 1, the factor  $\Gamma$  in Eq. (7.4) matches with the results derived in Refs. [58, 62] by setting  $\bar{N} = 1$ .

For the choice (7.4) the slow-roll parameters  $\hat{\epsilon}_t$  and  $\hat{s}_t$  in the transformed frame have the following relations

$$\hat{\epsilon}_t = -\hat{s}_t. \quad (7.5)$$

Since  $\hat{h} = \hat{N}\hat{H}$ , the slow-roll parameter  $\hat{\epsilon}_h$  reads

$$\hat{\epsilon}_h = \hat{\epsilon}_H - \hat{s}_t, \quad (7.6)$$

where

$$\hat{\epsilon}_H \equiv -\frac{1}{\hat{N}} \frac{\dot{\hat{H}}}{\hat{H}^2}. \quad (7.7)$$

Substituting Eqs. (7.2), (7.3), (7.5), and (7.6) into Eq. (7.1), it follows that

$$\hat{\mathcal{P}}_h(k) = \frac{2\hat{H}^2}{\pi^2 M_{\text{pl}}^2} [1 - 2(C+1)\hat{\epsilon}_H] |_{k=\hat{a}\hat{H}}, \quad (7.8)$$

where we used the fact that the condition  $\hat{c}_t k = \hat{a}\hat{h}$  translates to  $k = \hat{a}\hat{H}$  by using Eq. (7.2) with  $\hat{h} = \hat{N}\hat{H}$ . The spectrum (7.8) is equivalent to the next-to-leading order tensor power spectrum in GR [63]. As we will see in Sec. 7.2, the metric frame derived under the disformal transformation with the factors (7.4) can be regarded as the Einstein frame in which the function  $\hat{A}_4$  involves the term  $-M_{\text{pl}}^2/2$ , i.e., the Einstein-Hilbert term  $M_{\text{pl}}^2 R/2$  in the Horndeski Lagrangian (3.2).

Compared to the leading-order tensor spectrum  $\mathcal{P}_h^{\text{lead}}(k) = h^2/(4\pi^2 q_t c_t^3)$  in the original frame, the leading-order spectrum  $\hat{\mathcal{P}}_h^{\text{lead}}(k) = 2\hat{H}^2/(\pi^2 M_{\text{pl}}^2)$  in the Einstein frame depends on the Hubble parameter  $\hat{H}$  alone. This means that, even for very general inflationary models in the framework of GLPV theories, there is a frame in which the energy scale of inflation is directly known from the measurement of primordial gravitational waves. The disformal transformation provides us with the physical understanding that the amplitude of primordial tensor perturbations is intrinsically related to the expansion rate of the Universe in the Einstein frame.

Note that the leading-order tensor power spectrum in the Einstein frame was also found in Ref. [62] for the specific case with  $\Omega^2 = 1$ . We have further shown that the tensor spectrum in the Einstein frame matches with that appearing in GR even up to next-to-leading order in slow-roll for the more general transformation with  $\Omega^2 \neq 1$ .

Since  $\hat{N}^2 = \Omega^2 c_{tk}^2 = 8q_{tk}c_{tk}^3/M_{\text{pl}}^2$  for the choices (7.2) and (7.3), the scalar power spectrum (6.17) in the Einstein frame reads

$$\hat{\mathcal{P}}_\zeta(k) = \frac{q_{tk}c_{tk}^3}{q_{sk}c_{sk}^3} \frac{\hat{H}^2}{\pi^2 M_{\text{pl}}^2} [1 - 2(C+1)\hat{\epsilon}_H + 2(C+1)\hat{s}_t - C\hat{\epsilon}_s - (3C+2)\hat{s}_s] |_{\hat{c}_s k = \hat{a}\hat{h}}, \quad (7.9)$$

where we used Eqs. (5.13), (5.14), and (7.6).  $\hat{\mathcal{P}}_\zeta(k)$  depends not only on  $\hat{H}$  but also on the ratio  $q_{tk}c_{tk}^3/(q_{sk}c_{sk}^3)$ .

If  $c_s < c_t$ , the tensor-to-scalar ratio (6.33) in the Einstein frame evaluated at  $\hat{c}_t k = \hat{a} \hat{h}$  is given by

$$\hat{r} = 2 \frac{q_s c_s^3}{q_t c_t^3} \left[ 1 - 2(C+1)\hat{s}_t + C\hat{e}_s + (3C+2)\hat{s}_s + (2\hat{e}_H - 2\hat{s}_t + \hat{e}_s + 3\hat{s}_s) \ln \frac{c_s}{c_t} \right] \Big|_{k=\hat{a}\hat{H}}, \quad (7.10)$$

where we employed Eqs. (7.5) and (7.6).

If  $c_s > c_t$ , the evaluation of  $\hat{r}$  should be performed at  $\hat{c}_s k = \hat{a} \hat{h}$  [see Eq. (6.35)], such that

$$\hat{r} = 2 \frac{q_s c_s^3}{q_t c_t^3} \left[ 1 - 2(C+1)\hat{s}_t + C\hat{e}_s + (3C+2)\hat{s}_s + 2\hat{e}_H \ln \frac{c_s}{c_t} \right] \Big|_{\hat{c}_s k = \hat{a} \hat{h}}. \quad (7.11)$$

## 7.2 Background equations of motion in the Einstein frame and conditions for the red-tilted tensor power spectrum

From Eq. (7.8) the spectral index of the leading-order tensor power spectrum  $\hat{\mathcal{P}}_h^{\text{lead}}(k) = 2\hat{H}^2/(\pi^2 M_{\text{pl}}^2)$  in the Einstein frame is simply given by

$$\hat{n}_t^{\text{lead}} = -2\hat{e}_H. \quad (7.12)$$

The tensor spectrum is red-tilted ( $\hat{n}_t^{\text{lead}} < 0$ ) under the condition

$$\hat{e}_H > 0, \quad \text{i.e.,} \quad \dot{\hat{H}} < 0. \quad (7.13)$$

The same condition was also derived in Ref. [62] without specifying gravitational theories. In the following, we translate the condition  $\dot{\hat{H}} < 0$  explicitly in GLPV theories by considering the background equations of motion.

In the transformed frame, Eqs. (4.11) and (4.12) read

$$\hat{A}_2 - 6\hat{H}^2 \hat{A}_4 - 12\hat{H}^3 \hat{A}_5 + \hat{N} \left( \hat{A}_{2,\hat{N}} + 3\hat{H} \hat{A}_{3,\hat{N}} + 6\hat{H}^2 \hat{A}_{4,\hat{N}} + 6\hat{H}^3 \hat{A}_{5,\hat{N}} \right) = 0, \quad (7.14)$$

$$\hat{A}_2 - 6\hat{H}^2 \hat{A}_4 - 12\hat{H}^3 \hat{A}_5 - \frac{1}{\hat{N}} \left( \dot{\hat{A}}_3 + 4\dot{\hat{H}} \hat{A}_4 + 4\hat{H} \dot{\hat{A}}_4 + 12\hat{H} \dot{\hat{H}} \hat{A}_5 + 6\hat{H}^2 \dot{\hat{A}}_5 \right) = 0. \quad (7.15)$$

On using  $\hat{q}_t = \hat{L}_{,\hat{\mathcal{S}}}/(4\hat{N})$ , the choice (7.3) corresponds to  $\hat{L}_{,\hat{\mathcal{S}}} = M_{\text{pl}}^2/2$ , i.e.,

$$\hat{A}_4 = -\frac{M_{\text{pl}}^2}{2} - 3\hat{H} \hat{A}_5. \quad (7.16)$$

Since  $\hat{c}_t^2 = \hat{N}^2 \hat{\mathcal{E}}/\hat{L}_{,\hat{\mathcal{S}}}$  and  $\hat{\mathcal{E}} = \hat{B}_4 + \dot{\hat{B}}_5/(2\hat{N})$ , the choice (7.2) gives rise to another relation for  $\hat{B}_4$  and  $\hat{B}_5$ . However Eqs. (7.14) and (7.15) do not contain  $\hat{B}_4$ ,  $\hat{B}_5$  and their derivatives, so the background equations of motion are not affected by choosing  $\hat{c}_{tk}$  as Eq. (7.2). Substituting Eq. (7.16) into Eqs. (7.14) and (7.15), the terms  $3M_{\text{pl}}^2 \hat{H}^2$  and  $2M_{\text{pl}}^2 \dot{\hat{H}}$  arise from  $-6\hat{H}^2 \hat{A}_4$  and  $-4\dot{\hat{H}} \hat{A}_4$  respectively. Then we can express Eqs. (7.14) and (7.15) in the following forms

$$3M_{\text{pl}}^2 \hat{H}^2 = \hat{\rho}, \quad (7.17)$$

$$-2M_{\text{pl}}^2 \frac{1}{\hat{N}} \frac{d\hat{H}}{dt} = \hat{\rho} + \hat{P}, \quad (7.18)$$

where

$$\hat{\rho} \equiv -\hat{A}_2 - 6\hat{H}^3\hat{A}_5 - \hat{N} \left( \hat{A}_{2,\hat{N}} + 3\hat{H}\hat{A}_{3,\hat{N}} - 12\hat{H}^3\hat{A}_{5,\hat{N}} \right), \quad (7.19)$$

$$\hat{P} \equiv \hat{A}_2 + 6\hat{H}^3\hat{A}_5 - \frac{1}{\hat{N}} \left( \dot{\hat{A}}_3 - 12\hat{H}\dot{\hat{H}}\hat{A}_5 - 6\hat{H}^2\dot{\hat{A}}_5 \right). \quad (7.20)$$

The background equations of motion (7.17) and (7.18) correspond to those appearing in Einstein gravity. From these equations the effective energy density  $\hat{\rho}$  and the pressure  $\hat{P}$  obey the continuity equation

$$\frac{1}{\hat{N}} \frac{d}{dt} \hat{\rho} + 3\hat{H} (\hat{\rho} + \hat{P}) = 0. \quad (7.21)$$

From Eq. (7.18) the condition (7.13) translates to  $\hat{\rho} + \hat{P} > 0$ , i.e.,

$$\hat{N} \left( \hat{A}_{2,\hat{N}} + 3\hat{H}\hat{A}_{3,\hat{N}} - 12\hat{H}^3\hat{A}_{5,\hat{N}} \right) + \frac{1}{\hat{N}} \left( \dot{\hat{A}}_3 - 12\hat{H}\dot{\hat{H}}\hat{A}_5 - 6\hat{H}^2\dot{\hat{A}}_5 \right) < 0, \quad (7.22)$$

under which the tensor power spectrum is red-tilted. The explicit condition (7.22) is useful to confront inflationary models in the framework of GLPV theories with the observations of CMB.

## 8 Conclusions

On the flat FLRW background we have shown that the curvature perturbation  $\zeta$  and the tensor perturbation  $\gamma_{ij}$  are invariant under the disformal transformation  $\hat{g}_{\mu\nu} = \Omega^2(\phi)g_{\mu\nu} + \Gamma(\phi, X)\nabla_\mu\phi\nabla_\nu\phi$  by choosing the unitary gauge  $\phi = \phi(t)$ . This is the generalization of Ref. [42] in which the same property was also found for the transformation of the form  $\hat{g}_{\mu\nu} = \Omega^2(\phi)g_{\mu\nu} + \Gamma(\phi)\nabla_\mu\phi\nabla_\nu\phi$ . While the latter transformation preserves the structure of Horndeski theories, the former can deal with the transformation between more general theories beyond Horndeski, e.g., GLPV theories.

In unitary gauge the Lagrangian  $L$  of GLPV theories on the flat FLRW background is given by Eq. (3.5), which depends on the lapse  $N$ , the time  $t$ , and other three-dimensional geometric scalars  $K, \mathcal{S}, \mathcal{R}, \mathcal{U}$ . Under the disformal transformation (2.4), the structure of the action  $S = \int d^4x \sqrt{-g}L$  is preserved with the coefficients related to each other as Eqs. (3.17)-(3.22). The relations (3.23) and (3.24) imply that the transformation between Horndeski theories is required to satisfy the condition  $\Gamma_{,X} = 0$ , i.e.,  $\Gamma = \Gamma(\phi)$ .

Expanding the action (4.1) up to quadratic order in scalar and tensor perturbations, we have derived the corresponding second-order Lagrangian densities (4.23) and (4.28), respectively. Unlike Refs. [49, 50] we have explicitly taken into account the background value of the lapse  $N$ . This is important for studying the relations of physical quantities between the two metric frames connected under the disformal transformation. In GLPV theories we have presented explicit relations between the two frames for the quantities associated with the background and perturbation equations of motion. In particular, the quantities  $q_s, c_s^2, q_t, c_t^2$ , which are associated with conditions for the absence of ghosts and Laplacian instabilities, transform as Eqs. (5.13), (5.14), (5.20), and (5.21), respectively.

In Sec. 6 we have obtained the next-to-leading order inflationary power spectra of curvature and tensor perturbations as well as their spectral indices in the forms (6.17), (6.25), (6.27), and (6.30), respectively. For  $c_s < c_t$  the tensor-to-scalar ratio  $r$  is given by Eq. (6.33),

whereas for  $c_s > c_t$  it is expressed as Eq. (6.35). We have explicitly proved that the inflationary observables are invariant under the disformal transformation up to next-to-leading order in slow-roll.

In Sec. 7 we have identified the existence of the Einstein frame in which the next-to-leading order tensor power spectrum is in the same form as that in Einstein frame. The power spectra of tensor and scalar perturbations in this frame are given, respectively, by Eqs. (7.8) and (7.9). In the Einstein frame the function  $\hat{A}_4$  in GLPV theories has the relation  $\hat{A}_4 = -M_{\text{pl}}^2/2 - 3\hat{H}\hat{A}_5$ , in which case the Lagrangian (3.2) in Horndeski theories involves the Einstein-Hilbert term  $M_{\text{pl}}^2 R/2$ . In GLPV theories the condition under which the leading-order tensor power spectrum is red-tilted is characterized by Eq. (7.22).

Finally, we summarize the main results of our paper together with further possible applications.

- The next-to-leading order scalar and tensor power spectra derived in this paper are useful to place tight and precise constraints on a wide variety of single-field inflationary models in the framework of GLPV theories (along the lines of Refs. [39, 65]). In particular, the future possible detection of primordial gravitational waves will allow us to determine the inflationary Hubble parameter  $\hat{H}$  in the Einstein frame appearing in the tensor power spectrum (7.8).
- We have shown the invariance of curvature and tensor perturbations under the disformal transformation in the single-field inflationary scenario. In BD theories, it was further proved that other dimensionless cosmological observables—such as the redshift, the reciprocity relation, temperature anisotropies—are conformally independent of the chosen metric frames [34, 35]. We expect that the similar properties for the invariance of dimensionless observables would also hold for GLPV theories under the disformal transformation, but the detailed study will be necessary to understand the correspondence between physical quantities in different metric frames.
- In the context of dark energy we need to take into account additional matter fields to the Lagrangian. In such cases the propagation speeds of the scalar field  $\phi$  and matter are mixed each other even for the metric frame minimally coupled to matter [24, 26, 50, 64]. This non-trivial mixing can be understood by the disformal transformation to the Einstein frame under which the factor  $\Gamma$  involving the  $X$  dependence gives rise to a kinetic-type coupling of the scalar field with matter [22, 42]. It will be of interest to study the role of such a specific coupling and resulting observational consequences.

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